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TURBULENT BOUNDARY LAYER IN A GAS FLOW WITH HEAT TRANSFER AND A PRANDTL NUMBER DIFFERENT FROM UNITY

Yu. V. Lapin

The determination of surface friction and of heat transfer in a turbulent boundary layer during the movment of substances with large supersonic velocities is an urgent problem of aerodynamics. By now there have appeared a significant amount of articles devoted to this problem. However, the more or less full consideration of the influence of all factors (pressure gradients, radiation, and others) on the dynamics and heat characteristics of a boundary layer remains very intricate.

The existing methods of calculation of turbulent boundary layer in a gas with heat transfer for both the case of a flow past a flat plate and for the flow with pressure gradients are based on the transfer to gas dynamics of the formulas of semiempirical theories of turbulence. This article is in this sense no exception. The solution is obtained for the flow of gas with a moderate pressure gradient and with an arbitrary temperature distribution at the wall since the Karman semiempirical theory is used. The influence of the pressure gradient is calculated only across the external flow. The temperature

range is examined in which the Prandtl number and the specific heat can be considered constant, regardless of temperature.

In this article we obtain the analytical expression for the parameter $H = \delta^*/\delta^{**}$ depending on the conditions at the wall and at the exterior boundary layer. The final formula obtained in this article for the calculation of friction does not require the use of various types of auxiliary graphs and tables, as in other existing methods. This advantage can prove to be beneficial in a number of cases. The calculation of heat transfer is based on Reynold's analogy.

1. Basic equations

The differential equations of continuity, of momentum and of average stationary plane kinetic energy in a turbulent boundary layer have the form:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0; \quad (1)$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left[(\mu + \varepsilon)\frac{\partial u}{\partial y}\right]; \quad (2)$$

$$\rho u \frac{\partial T_0}{\partial x} + \rho v \frac{\partial T_0}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \varepsilon)\frac{\partial T_0}{\partial y}\right] + \left(\frac{1}{Pr} - 1\right)\frac{\partial}{\partial y}\left(\mu \frac{\partial T}{\partial y}\right). \quad (3)$$

where x, y and u, v — are respectively the coordinates and the components of the velocity along the surface and normal to it;

ρ = density of the gas;

p = pressure;

μ and ε are respectively the coefficients of dynamic and of eddy viscosity;

T_0 = stagnation temperature in a boundary layer;

T = temperature of the gas in a boundary layer;

c_p = specific heat with constant pressure;

J = the mechanical equivalent of heat;

g = the accelerating force of gravity;

$Pr=\frac{g\mu c_p}{\lambda}$ = the Prandtl number;

λ = the coefficient of heat transfer of the gas.

The boundary conditions are:

$$\left.\begin{aligned} u=v=0,\ T=T_w(x) \text{ when } y=0; \\ u=U_e(x),\ T=T_e(x) \text{ when } y=\infty. \end{aligned}\right\} \tag{4}$$

We will consider the external flow as isoentropic, i.e. the total pressure p_{eo} and stagnation temperature T_{eo} are constant.

Outside of the boundary layer Bernoulli's equation holds true:

$$\frac{dp_e}{dx}=-\varrho_e U_e\frac{dU_e}{dx}. \tag{5}$$

From equations (1), (2), and (5) and boundary conditions (4) the integral relation of momentum can be obtained by the usual method:

$$\frac{d\bar{\delta}^{**}}{d\bar{x}}+P(\bar{x})\bar{\delta}^{**}=\frac{1}{h^2}\cdot\frac{T_e}{T_w}. \tag{6}$$

Here

$$P(\bar{x})=\frac{2+H-M_e^2}{M_e\left(1+\frac{k-1}{2}M_e^2\right)}\cdot\frac{dM_e}{d\bar{x}}, \tag{7}$$

where

$$H=\frac{\delta^*}{\delta^{**}}: \tag{8}$$

$$\delta^{**}=\int_0^\infty\frac{\varrho u}{\varrho_e U_e}\left(1-\frac{u}{U_e}\right)dy \text{ — is the thickness of the momentum loss;} \tag{9}$$

$$\delta^*=\int_0^\infty\left(1-\frac{\varrho u}{\varrho_e U_e}\right)dy \text{ — is the displacement thickness,} \tag{10}$$

$$h=\frac{U_e}{v_*}=\sqrt{\frac{2T_e}{c_f T_w}}. \tag{11}$$

where

$$v_* = \sqrt{\frac{\tau_w}{\varrho_w}};\ \tau_w = \mu_w\left(\frac{\partial u}{\partial y}\right)_w$$ — is the friction at the wall;

$$c_f = \frac{2\tau_w}{\varrho_e U_e^2}$$ — is the coefficient of friction.

Here and throughout the article, a dash will designate the dimensionless values ($\overline{x} = \frac{x}{L}$, $\overline{\delta}^{**} = \frac{\delta^{**}}{L}$, $\overline{u} = \frac{u}{U_e}$, ...), obtained by division of the dimensional value by a selected scale (for example, L = airfoil chord).

2. Relation Between Velocity Profile and Temperature Profile

In the laminar sublayer, the expression for stagnation temperature will be sought as a series in powers of u; in view of the smallness of u (separation close to the wall) we will confine ourselves to three terms

$$T_0 = a_0 + a_1 u + a_2 u^2. \tag{12}$$

We will note that $T_0 = T_w$ when u = 0 and, consequently,

$$a_0 = T_w. \tag{12a}$$

Having differentiated equality (12) with respect to y and having taken the obtained expression when y = 0, we will find that

$$a_1 = -\frac{q_w Pr}{g c_p \tau_w}. \tag{12b}$$

For the determination of the coefficient a_2 we will twice differentiate equality (12) with respect to y and we will use equations (2) and (3) where y = 0; then after simple conversion we will obtain

$$a_2 = \frac{1 - Pr}{2gJc_p} - \frac{Prq_w\mu_w}{2c_p g\tau_w^3}\cdot\frac{dp}{dx}. \tag{12c}$$

By the examined mean pressure gradients, the second term in equality (12) proves to be negligible in value and can be omitted in the future.

We will introduce with respect to h the value h_t, expressed by the equality

$$h_t = \sqrt{\frac{T_e}{St\,T_w}}. \tag{13}$$

where the Stanton number

$$St = \frac{q_w}{\varrho_e U_e g c_p (T_t - T_w)} \tag{14}$$

represents the dimensionless coefficient of heat transfer, $q_w = -\left(\lambda \frac{\partial T}{\partial y}\right)_w$ is the heat flow, T_t is the equilibrium temperature of the flow surface without heat transfer.

Using equality (11) and the expression for stagnation temperature

$$T_0 = T + \frac{k-1}{2} M_e^2 T_e \bar{u}^2, \tag{15}$$

we will convert the relation (12) to:

$$\frac{T}{T_w} = 1 - Pr\omega\bar{u} - Pr\gamma\bar{u}^2, \tag{16}$$

where

$$\omega = \left(\frac{h}{h_t}\right)^2 \left(1 - \frac{T_t}{T_w}\right), \quad \gamma = \frac{k-1}{2} M_e^2 \frac{T_e}{T_w}. \tag{17}$$

In the turbulent core we will use Reynolds analogy, which in the case of a compressible gas is expressed by the formula [1]:

$$\frac{q}{\tau} - g c_p \frac{\partial T_0}{\partial u}. \tag{18}$$

If we simply assume the constancy of the friction force $\tau = \tau_\omega$ and of heat flow $q = q_\omega$ across the boundary layer and integrating (18) we will have:

$$T_0 = -\frac{q_w}{c_p \tau_w} u + C(x). \qquad (19)$$

Determining C(x) from the condition at the boundary of the laminar sublayer we will obtain after conversion the following relation of temperature profile with velocity profile in a turbulent core:

$$\frac{T}{T_w} = 1 + \beta - \omega \bar{u} - \gamma \bar{u}^2, \qquad (20)$$

where

$$\beta = (1 - Pr)(\omega \bar{u}_{1t} + \gamma \bar{u}_{1t}^2), \qquad (21)$$

and $\bar{u}_{1t}$ is the dimensionless velocity at the boundary of the laminar heat sublayer.

From equality (20) and boundary condition $T = T_e$ where $\bar{u} = 1$ we will obtain the expression for ω:

$$\omega = \frac{1 - \frac{T_{e0}}{T_w} + \gamma \bar{u}_{1t}^2 (1 - Pr)}{1 - (1 - Pr) \bar{u}_{1t}}. \qquad (22)$$

The previously obtained relations between temperature profile in a laminar sublayer (16) and in a turbulent core (20) in the case of a flat plate ($\frac{dp}{dx} = 0$) and Pr = 1 convert to the known Crocco relation:

$$\frac{T}{T_w} = 1 - \left(1 - \frac{T_{e0}}{T_w}\right) \bar{u} - \frac{k-1}{2} M_e^2 \frac{T_e}{T_w} \bar{u}^2. \qquad (23)$$

From the equalities (16), (20) and the equation of the component

$$p = \varrho R T \qquad (24)$$

the expression for surface gas in a laminar sublayer is obtained:

$$\frac{\varrho}{\varrho_e} = \frac{1 + \beta - \omega - \gamma}{1 - Pr\,\omega \bar{u} - Pr \gamma \bar{u}^2} \qquad (25)$$

and in a turbulent core:

$$\frac{\varrho}{\varrho_e} = \frac{1+\beta-\omega-\gamma}{1+\beta-\omega\bar{u}-\gamma\bar{u}^2}. \tag{26}$$

3. Determination of Integral Thicknesses of a Boundary Layer

We will show the method of determining the integral thicknesses by the example of the calculation of momentum loss thickness δ^{**}.

In the expression for momentum loss thickness (19) we will convert to the universal coordinates:

$$\varphi = \frac{u}{v_*}, \quad \eta = \frac{y v_*}{\nu_w}, \tag{27}$$

then we will have:

$$\delta^{**} = \frac{\nu_w h}{U_e} \int_0^\infty \frac{\varrho}{\varrho_e} \cdot \frac{\varphi}{h}\left(1 - \frac{\varphi}{h}\right) d\eta. \tag{28}$$

We can present equality (28) also in the form:

$$\delta^{**} = \frac{\nu_w h^2}{U_e} \int_0^1 \frac{\varrho}{\varrho_e} \bar{u}(1-\bar{u})\dot{\eta} d\bar{u}, \tag{29}$$

where $$\dot{\eta} = \frac{d\eta}{d\varphi}.$$

The relations of densities $\frac{\varrho}{\varrho_e}$ in a laminar sublayer and in a turbulent core are expressed by equalities (25) and (26) respectively. Thus, for the determination of δ^{**} it is necessary to have the relationship of the value η to dimensionless velocity $\bar{u}$.

In a turbulent core for the determination of η we will use the Karman relation:

$$\tau = \varrho \varkappa^2 \frac{u'^4}{u''^2}, \tag{30}$$

where $\varkappa$ is the constant of turbulence, the value of which is taken equal to $\varkappa = 0.4$ as in the case of an incompressible fluid; the prime in equality (30) designates a derivative with respect to y.

Converting in (30) to universal coordinates and bearing in mind

equality (20) after simple conversions we will obtain the equation

$$\frac{\varphi''}{\varphi'^2} = -\frac{\varkappa}{\sqrt{1+\beta-\omega\bar{u}-\gamma\bar{u}^2}}. \tag{31}$$

The minus sign appears after the root extraction because $\varphi'' < 0$, if the influence of the pressure gradient on the velocity profile is not considered.

In equality (31) let us interchange the argument and the function, then we will obtain the equation:

$$\frac{\dot{\eta}'}{\dot{\eta}} = \frac{\varkappa}{\sqrt{1+\beta-\omega\bar{u}-\gamma\bar{u}^2}}, \tag{32}$$

the equality

$$\dot{\eta} = C\exp\left(\frac{\varkappa h}{\sqrt{\gamma}}\arcsin\frac{\sqrt{\gamma}\,\bar{u}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}\right). \tag{33}$$

is used for the first integral of (32).

For the determination of the constant C, we assume that this value is the same as in the case of incompressible gas on the flat plate, i.e. $C = \frac{1}{f} = \varkappa\alpha$, where α is the turbulent constant, the value of which, as in the case of incompressible gas, is taken to equal $\alpha = 11.5$. In other words, it is assumed that the value of the derivative $\dot{\eta}$ in the case of a turbulent core does not depend on compressability and gradient pressure. We note that the assumption of the non-dependence of the value η on compressability with no pressure gradient agrees with the experiment.

The assumption leads to the following result:

$$\dot{\eta} = \frac{1}{f}\exp\left[\frac{\varkappa h}{\sqrt{\gamma}}\left(\arcsin\frac{\sqrt{\gamma}\,\bar{u}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}} - \arcsin\frac{\sqrt{\gamma}\,\bar{\alpha}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}\right)\right]. \tag{34}$$

where $\bar{\alpha} = \frac{\alpha}{h}$.

The integral in the right-hand part of equality (29), strictly speaking, should be divided into two parts: $0 < \bar{u} < \bar{\alpha}$ and $\bar{\alpha} < \bar{u} < 1$ and could substitute their own value for the values η and $\frac{\rho}{\rho_e}$ in each of these. However, in view of the relative thinness of the laminar sublayer, we can omit the first part having continued the second up to the wall.

Thus, excluding $\dot{\eta}$ with the help of equality (34) and substituting the value ρ/ρ_e from (26), we will have the following expression for the momentum loss thickness:

$$\delta^{**} = \frac{(1+\beta-\omega-\gamma)h^2\nu_w}{jU_e}\int_0^1 \frac{\bar{u}(1-\bar{u})}{1+\beta-\omega\bar{u}-\gamma\bar{u}^2} \times$$

$$\times \exp\left[\frac{\varkappa h}{\sqrt{\gamma}}\left(\arcsin\frac{\sqrt{\gamma}\bar{u}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}} - \arcsin\frac{\sqrt{\gamma}\bar{\alpha}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}\right)\right]d\bar{u}. \tag{35}$$

Introducing a new variable

$$\psi = \frac{1}{\sqrt{\gamma}}\arcsin\frac{\sqrt{\gamma}\bar{u}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}, \tag{36}$$

we will obtain

$$\delta^{**} = \frac{(1+\beta-\omega-\gamma)\nu_w h^2\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}{jU_e\gamma}\int_{\psi_w}^{\psi_e}\frac{\exp[\varkappa h(\psi-\psi_1)]}{\cos\sqrt{\gamma}\psi} \times$$

$$\times\left[\sin\sqrt{\gamma}\psi-\sin\sqrt{\gamma}\psi_w\right]\left[\sin\sqrt{\gamma}\psi_e-\sin\sqrt{\gamma}\psi\right]d\psi, \tag{37}$$

where

$$\left.\begin{aligned}
\psi_e &= \frac{1}{\sqrt{\gamma}}\arcsin\frac{\sqrt{\gamma}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}},\\
\psi_w &= \frac{1}{\sqrt{\gamma}}\arcsin\frac{\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}};\\
\psi_1 &= \frac{1}{\sqrt{\gamma}}\arcsin\frac{\sqrt{\gamma}\bar{\alpha}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}.
\end{aligned}\right\} \tag{38}$$

The value $\varkappa h$, as is obvious from its determination, essentially exceeds unity, therefore for the calculation of integral (37) we use a general representation of the integral containing an exponential function in the form of an asymptotic series which is obtained as a result of integration by parts:

$$\int e^{\varkappa h\psi} f(\psi)\,d\psi = \frac{e^{\varkappa h\psi}}{\varkappa h}\left[f(\psi) - \frac{1}{\varkappa h} f'(\psi) + \frac{1}{\varkappa^2 h^2} f''(\psi) - \ldots\right]. \qquad (39)$$

Fulfilling in (37) the integration accurately to the term containing $\varkappa^3 h^3$ in the denominator and representing ψ_1 approximately as

$$\psi_1 = \psi_w + \frac{a}{h}. \qquad (40)$$

we will obtain the following expression for the momentum loss thickness:

$$\delta^{**} = \frac{e^{-\varkappa a} v_w (1+\beta-\omega-\gamma)}{f\varkappa^2 U_e} e^{\varkappa h(\psi_e-\psi_w)}\left(1 - \frac{2+2\beta-1{,}5\omega-\gamma}{\varkappa h\sqrt{1+\beta-\omega-\gamma}}\right). \qquad (41)$$

The following expression for the displacement thickness can be analogously obtained:

$$\delta^{*} = \frac{e^{-\varkappa a} v_w (1+\beta+\gamma)}{f\varkappa^2 U_e} e^{\varkappa h(\psi_e-\psi_w)} \times$$
$$\times\left[1 - \frac{\gamma\left(3+3\beta-1{,}5\omega-\gamma+\frac{\omega}{2\gamma}+\frac{\omega\beta}{2\gamma}\right)}{\varkappa h(1+\beta+\gamma)\sqrt{1+\beta-\omega-\gamma}}\right]. \qquad (42)$$

where

$$\psi_e - \psi_w = \frac{1}{\sqrt{\gamma}}\left(\arcsin\frac{\sqrt{\gamma}+\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}} - \arcsin\frac{\frac{\omega}{2\sqrt{\gamma}}}{\sqrt{1+\beta+\frac{\omega^2}{4\gamma}}}\right). \qquad (43)$$

If we set up the ratio of the equalities (42) and (41) we will obtain the expression for H:

$$H = \frac{(1+\beta+\gamma)\left[1 - \frac{\gamma\left(3+3\beta-1{,}5\omega-\gamma+\frac{\omega}{2\gamma}+\frac{\omega\beta}{2\gamma}\right)}{\varkappa h(1+\beta+\gamma)\sqrt{1+\beta-\omega-\gamma}}\right]}{(1+\beta-\omega-\gamma)\left(1-\frac{2+2\beta-1{,}5\omega-\gamma}{\varkappa h\sqrt{1+\beta-\omega-\gamma}}\right)}. \tag{44}$$

In a limiting case of the flow of an incompressible gas without heat transfer we will obtain the following expression for H:

$$H = \frac{1}{1-\frac{2}{\varkappa h}}. \tag{45}$$

which with Reynolds numbers $10^6 \div 10^7$ yields the value $H \approx 1.3$, which agrees with the experiment.

4. Solution of Integral Equation of Momenta

Introducing a new variable in the equation of momenta (6)

$$\zeta = \bar{\delta}^{**} \exp\left[\int_{\bar{x}_H}^{\bar{x}} P(\bar{x})\, d\bar{x}\right]. \tag{46}$$

we will reduce it to

$$\frac{d\zeta}{d\bar{x}} = \frac{e^N}{h^2} \cdot \frac{T_e}{T_w}, \tag{47}$$

where

$$N = \int_{\bar{x}_H}^{\bar{x}} P(\bar{x})\, d\bar{x}, \tag{48}$$

and $\bar{x}_H$ is the dimensionless coordinate of the beginning of the turbulent part of the boundary layer.

Integrating equation (47) and reverting δ^{**} we will obtain:

$$\bar{\delta}^{**} = e^{-N}\left(\int_{\bar{x}_H}^{\bar{x}} \frac{e^N}{h^2} \cdot \frac{T_e}{T_w}\, d\bar{x} + C\right). \tag{49}$$

If in (49) in place of $\bar{\delta}^{**}$ we substitute its value from equality (41) we will have:

$$F(\bar{x})\left(1 - \frac{2+2\beta-1{,}5\omega-\gamma}{\varkappa h\sqrt{1+\beta-\omega-\gamma}}\right) e^{\varkappa h(\varphi_e-\varphi_w)} = e^{-N}\left(\int_{\bar{x}_H}^{\bar{x}} \frac{e^N}{h^2} \cdot \frac{T_e}{T_w}\, d\bar{x} + C\right). \tag{50}$$

where

$$\bar{F}(x) = \frac{e^{-\varkappa a}(1+\beta-\omega-\gamma)\nu_w}{j\varkappa^2 U_e L}. \tag{51}$$

Equation (50) permits us to use the method of successive approximation for the determination of h. Carrying out the conversion, we will obtain:

$$h = \frac{1}{\varkappa(\psi_e-\psi_w)} \ln \frac{\int\limits_{\bar{x}_\kappa}^{\bar{x}} \frac{e^N}{h^2} \cdot \frac{T_e}{T_w} d\bar{x} + C}{e^N F(\bar{x})\left(1 - \frac{2}{\varkappa h} \cdot \sqrt{\frac{2\beta-1.5\omega-\gamma}{1+\beta-\omega-\gamma}}\right)}. \tag{52}$$

The constant of integration C is determined, as usual, from the condition of the congruence of momentum loss thicknesses at the point of conversion of the laminar boundary layer to the turbulent boundary layer. Upon completion of the turbulent boundary layer, the constant of integration C is reduced to zero, and equality (52) is converted to

$$h = \frac{1}{\varkappa(\psi_e-\psi_w)} \ln \frac{\int\limits_{0}^{\bar{x}} \frac{e^N}{h^2} \cdot \frac{T_e}{T_w} d\bar{x}}{e^N F(\bar{x})\left[1 - \frac{2}{\varkappa h} \sqrt{\frac{2\beta-1.5\omega-\gamma}{1+\beta-\omega-\gamma}}\right]}. \tag{53}$$

It is natural to take the value h calculated for a flat plate as a zero approximation.

5. Calculation of Heat Transfer

Using the equalities (17) and (22), we will find the expression for the parameter $\left(\frac{h}{h_t}\right)^2$:

$$\left(\frac{h}{h_t}\right)^2 = \frac{2St}{c_f} = \frac{1-\frac{T_{en}}{T_w}+(1-Pr)\gamma\bar{u}^2_{1t}}{\left(1-\frac{T_t}{T_w}\right)[1-(1-Pr)\bar{u}_{1t}]}. \tag{54}$$

The numerator of the right-hand part of equality (54) can be considered equal to $1 - \frac{T_t}{T_\omega}$, although, strictly speaking, it differs somewhat from that remainder since, in the conversion to the case of $q_\omega = 0$, the value $\overline{u}_{1t}$ changes; however, the influence of this change is negligible. Therefore, with great accuracy the equality (54) can be written as:

$$\frac{2St}{c_f} = \frac{1}{1-(1-Pr)\bar{u}_{1t}}. \qquad (55)$$

This formula agrees in form with the Prandtl formula [2], obtained by him for the flow of an incompressible fluid in a tube.

The value of a dimensionless velocity at the boundary of a heat laminar sublayer is expressed by the equality:

$$\bar{u}_{1t} = \frac{\alpha_t}{h}. \qquad (56)$$

where $\alpha_t = \frac{\alpha}{\sqrt{Pr}}$ is the constant of a heat laminar sublayer.

We note that formula (55) yields the results agreeing with the A. R. Colburn experimental data [3], who found that with small velocities and minor temperature differences the relation of the Stanton number to one-half of the friction is expressed by the ratio:

$$\frac{2St}{c_f} = Pr^{-\frac{2}{3}}. \qquad (57)$$

6. Calculation Example.

Calculations of friction and heat transfer on a wing profile in a supersonic flow were carried out by the stated method. The profile described by the formula $\overline{y} = 0.16\,\overline{x}\,(1-\overline{x})$, at angle of attack $\theta = 3°$ was taken as the initial profile.

The distribution of numbers M_e on the profile were calculated with respect to the conventional formula of the theory of a profile in

supersonic flow.

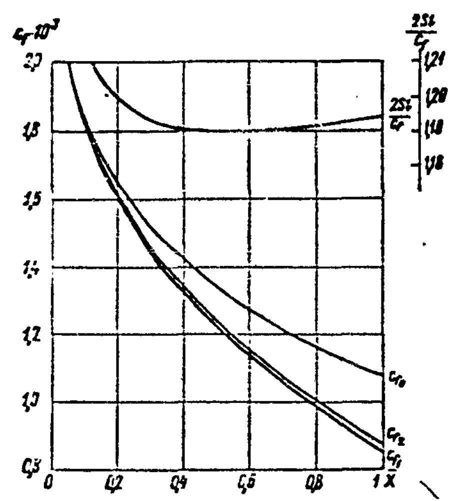
$$\frac{M_e}{M_\infty} = 1 - \frac{1 - 0.2M_\infty^2}{\sqrt{M_\infty^2 - 1}}\theta_p +$$

$$+ \frac{0.04M_\infty^6 + 0.14M_\infty^4 - 0.4M_\infty^2 - 0.5}{(M_\infty^2 - 1)^2}\theta_p^2,$$

where θ_p is the local angle of attack.

In the external flow, conditions at a height of flight H = 20 km where $M_\infty = 6$ were used. The temperature at the wall was taken to be $T_w = 700°$ K.

The results of the calculation of the coefficient of friction c_f and of the parameter $2St/c_f$ are presented in the diagram. As appears in the diagram, for practical purposes we can be satisfied with the results of the second approximation.



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